Tables of Pure Reciprocants to the Weight 8.

By Prof. Cayley.

In the tabulation of Pure Reciprocants it is convenient to write $a = 1$; we thus have for all the reciprocants of a given weight a single column of literal terms which (as in the Seminvariant Tables) I arrange in alphabetical order AO, and the several reciprocants have then each of them its own column of numerical coefficients: the form of the table is thus similar to that of the seminvariant table, the only difference being that for reciprocants the final terms are not in general power-enders: as in the seminvariant table, the columns of the table are arranged *inter se* with their final terms in AO. As remarked in my paper, "Corrected Seminvariant Tables for the Weights 11 and 12," A. M. J., t. XIV (1892), pp. 195–200, it is not in every case the top term of a column which should be regarded as the initial term; but to the extent 8 to which the reciprocant tables are here carried this remark has no application.

I recall that the notation is the modified one employed by Halphen, and by Sylvester in his 12th and subsequent lectures, viz. a, b, c, d, ... denote $\frac{1}{2}\frac{d^2y}{dx^2}$, $\frac{1}{6}\frac{d^3y}{dx^3}$, $\frac{1}{24}\frac{d^4y}{dx^4}$, $\frac{1}{120}\frac{d^5y}{dx^5}$, ... respectively. As already noticed, a is put $= 1$, but it is to be in the several terms restored in the proper powers so as to obtain for the reciprocant a homogeneous expression of a degree equal to the original degree of the final term; thus $d - 3bc + 2b^3$ is to be read as standing for $a^2d - 3abc + 2b^3$.

The ultimate verification of the expression for a pure reciprocant consists (as is known) in its annihilation by the operator

$$V = 2a^2\partial_b + 5ab\partial_c + (6ac + 3b^2)\partial_d + (7ad + 7bc)\partial_e + (8ae + 8bd + 4c^2)\partial_f + \text{etc.,}$$

or say

$$V = 2\partial_b + 5b\partial_c + (6c + 3b^2)\partial_d + (7d + 7bc)\partial_e + (8e + 8bd + 4c^2)\partial_f + \text{etc.;}$$

thus for the reciprocant $50e - 175bd + 28c^2 + 105b^2c$, the result obtained is

$$2(-175d + 210bc) + 5b(56c + 105b^2) + (6c + 3b^2)(-175b) + (7d + 7bc)(50),$$

or collecting, this is

d	-350		$+350$	± 350
bc	$+420 + 280 - 1050$		$+350$	± 1050
b^3	$+525 -$	525		± 525

$= 0$, as it should be.

The tables are

c	$+4$
b^2	-5
	$+4$
	-5

d	$+1$
bc	-3
b^3	$+2$
	± 3

e	$+50$	
bd	-175	
c^2	$+28$	$+16$
b^2c	$+105$	-40
b^4		$+25$
	$+183$	$+41$
	-175.	-40

f	$+10$	
be	-40	
cd	-12	$+4$
b^2d	$+65$	-5
bc^2	$+16$	-12
b^3c	-39	$+23$
b^5		-10
	± 91	± 27

g	$+14$			
bf	-63			
ce	-1350	$+800$		
d^2	$+1470$	-875	$+125$	
b^2e	$+1782$	-1000		
bcd	-4158	$+2450$	-750	
c^3	$+2130$	-1344	$+256$	$+64$
b^3d			$+500$	
b^2c^2		$+35$	$+165$	-240
b^4c			-300	$+300$
b^6				-125
	$+5576$	$+3250$	± 1018	$+364$
	-5508	-3254		-365

h	$+7$			
bg	-35			
cf	-539	$+560$		
de	$+605$	-650	$+50$	
b^2f	$+735$	-700		
bce	$+306$	-290	-150	
bd^2	-2135	$+2275$	-175	
c^2d	$+1001$	-1036	$+28$	$+16$
b^3e	-1485	$+1500$	$+100$	
b^2cd	$+3465$	-3710	$+630$	-40
bc^3	-1295	$+1988$	-84	-48
b^4d			-350	$+25$
b^3c^2		$+63$	-259	$+152$
b^5c			$+210$	-155
b^7				$+50$
	$+6119$	± 6386	± 1018	± 243
	-5489			

i	+ 420					
bh	− 2310					
cg	− 32704	+ 1176				
df	+ 57750	− 8085	+ 20433			
e^2	− 20460	+ 7040	− 21542	+ 625		
b^2g	+ 45500	− 1470				
bcf	− 28392	+ 18963	− 61299			
bde	− 90900	− 16940	+ 69062	− 4375		
c^2e	+103740	− 27160	+ 80248	− 49700	+ 3200	
cd^2	− 38320	+ 26460	− 85554	+ 55125	− 3500	+ 500
b^3f	− 69615	− 9555	+ 40866			
b^2ce	+ 83538	+ 28098	−106218	+128625	− 8000	
b^2d^2	+ 92820	+ 12740	− 54782	− 61250	+ 4375	− 625
bc^2d	−102102	− 52822	+191590	−156800	+ 9800	−3000
c^4		+ 21560	− 73304	+ 84868	− 5376	+1024
b^4e		− 378	− 78750	+ 5000		
b^3cd		+ 1176	+183750	−12250	+5750	
b^2c^3			−102165	+ 6580	− 620	
b^5d					−2500	
b^4c^2				+ 175	−2025	
b^6c					+1500	
b^8						

c^4	+ 256					
b^2c^3	−1280					
b^4c^2	+2400					
b^6c	−2000					
b^8	+ 625					

$+383768$ $+116037$ $+403375$ $+452993$ $+29130$ $+8774$ $+3281$

-384803 -116032 -403077 -453040 -29126 -8750 -3280

I remark that in the last of these tables the first column, say $i \infty bc^2d$, which ends in bc^2d, is a more simple form than Sylvester's P_8, $= i \infty c^4$, (A. M. J., t. IX, p. 35) which ends in c^4; P_8 is in fact a linear combination, first col. + 6 second col. of the first and second columns of the table: the second column, say $cg \infty c^4$ is Sylvester's (a^2cg), t. IX, p. 124.